DEVELOPMENT OF FLOOD ROUTING MODELS FOR WANG RIVER BASIN

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Received Date: March 31, 2014

Abstract

Nowadays, severe flooding frequently occurs in various parts of Thailand resulted from changes in climatic condition and land use patterns. The flooding has caused great damages to properties and lives and affects country economy. Experience from the most severe flooding in the northern and central regions of Thailand in the year 2011 reveals that reliable flood warning system is still lagging. For flood warning purpose, it is necessary to have an accurate flood routing system. This study is aimed at developing mathematical models for flood routing so as to provide data for flood warning. Two different models are developed, i.e., kinematic overland flow model and kinematic stream flow model. The finite element method with Galerkin’s weighted residual technique is used in model development. The second-order Runge-Kutta method is applied to solve the set of differential equations obtained from finite element formulation. The developed models are applied to simulate flows in the Wang river basin in the northern region of Thailand during July 1 – October 31, 2011 when severe flooding occurred in this region. Model calibration is made by adjusting some parameters in the models and comparing the obtained results with measured data recorded by RID at 5 stream flow gauge stations along the Wang river. For correlation analysis, three statistical indices are determined, these include coefficient of determination, $R^2$, Nash–Sutcliffe model efficiency coefficient, NSE, and coefficient of variation of the root mean square error, CV(RMSE). It is found that the model results at the upstream portion of the river satisfactorily agree with the observed data, with the values of $R^2$ greater than 0.55 and CV(RMSE) less than 0.57. For the downstream portion of the river, there are remarkable differences between the model results and the observed data. The values of $R^2$ are less than 0.35, CV(RMSE) greater than 0.76, and the NSE values are less than 0.16. This might be due to some errors in the input data, including rainfall pattern, topography, land use, river cross-sectional area, and water seepage along the river. More detailed field investigation and model calibration are still needed.

Keywords: Finite element method, Flood routing, Hydrological model, Wang River Basin

Introduction

Rapid population growth and urbanization in Thailand in the past few decades have created high demand of land resources. As a result, deforestation has been undertaken in many parts of the country, especially in mountainous areas. It was reported that the whole forest area in Thailand had decreased from 53.3% of the country area in 1961 to 30.9% in 2006 [1]. In addition, land use patterns have been gradually changed. These activities together with some other factors including global warming which has affected rainfall and stream flow patterns in various regions [2]. Flash flood and drought occur very often nowadays. In 2011, flooding occurred in many regions of Thailand for prolonged period which caused great damage to lives and economy of the country [3]. One factor which caused great damage was the lacking of information on flood magnitude and time of occurrence [4].
So, flood warning system is needed for Thailand and it is necessary to have reliable information on flood magnitudes in downstream regions, especially in urbanized and industrial areas.

Several flow routing and water resources management models have been developed in the past few decades. These include HEC programs, such as HEC-1, HEC-2, HEC-HMS, HEC-RAS, etc., developed by Hydrologic Engineering Center of the U.S. Army Corps of Engineers [5], Watershed Modeling System (WMS) developed by the Environmental Modeling Research Laboratory of Brigham Young University [6], Soil and Water Assessment Tool (SWAT) developed by USDA Agricultural Research Service and Texas A & M AgriLife Research [7], CASC2D developed by Texas A & M University and U.S. Bureau of Reclamation [8], etc. Some of these models were applied to investigate river flow and flood propagation in the Chao Phraya river basin and its main tributaries, namely Ping, Wang, Yom and Nan rivers. These models were developed by using finite difference method for numerical approximation.

Besides the finite difference method, finite element method has been found to be a very effective numerical technique for solving partial differential equations. It has some advantages over the finite difference method, especially in analyzing problems over complicated domains with irregular boundaries. In the finite element method, the study domain is divided into elements of which several types, shapes and sizes can be used [9]. Moreover, the procedure in finite element model development is well defined compared with the finite difference method [10]. With these advantages, the finite element method becomes more popular nowadays. In this study, the finite element method is used in developing flood routing models for the Wang river basin.

The objective of this study is to develop flood routing models for Wang river basin using finite element method. The main purposes are to determine the applicability and accuracy of the finite element method in flood routing and to obtain reliable flood forecasting models for Wang river which can be used to determine flood magnitude and time of occurrence when heavy storms occur in the river basin. The obtained results will be beneficial to those agencies which are responsible for flood management and warning.

Method

The finite element method is used for model development. The Galerkin’s weighted residual technique is employed to convert basic governing equations which are in the form of partial differential equations to sets of first-order differential equations. With given initial and boundary conditions, these sets of differential equations are solved by using the second-order Runge-Kutta method to obtain the values of flow rates at various nodal points identified in the study area which includes watershed areas and streams.

Model Formulation

In this study, 2 types of models are developed for Wang river basin, i.e. kinematic overland flow model for computation of overland flow due to excess rainfall on catchment areas of the Wang river, and kinematic stream flow model for computation of flow in the Wang river.

Kinematic Overland Flow Model

This model is based on the following Equations [11]:

\[
\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = i - f
\]

(1)
Discharge-water depth relationship: \[ q = \alpha h^m = \frac{\sqrt{S}}{n_o} h^{\frac{5}{3}} \] (2)

in which \( h \) is overland water depth (m); \( q \) is overland flow per unit width (m\(^3\)/s.m); \( q_x \) and \( q_y \) are overland flow rates per unit width (m\(^3\)/s.m) in the x- and y-directions, respectively; \( i \) is rainfall intensity (m/s); \( f \) is rate of water loss due to infiltration and evaporation (m/s); \( \alpha \) is conveyance factor \( = \sqrt{S} n_o \); \( S \) is overland flow slope; \( n_o \) is an effective roughness parameter for overland flow; and \( m \) is a constant \( = 5/3 \) from Manning's equation.

The overland flow rates \( q_x \) and \( q_y \) can be expressed in terms of \( q \) as follow:

\[ q_x = q \cos \theta \] and \[ q_y = q \sin \theta \] (3)

where \( \theta \) is the angle between flow direction and the x-axis.

Therefore, the continuity equation can be written as

\[ \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} \cos \theta + \frac{\partial q}{\partial y} \sin \theta = i - f \] (4)

Replace \( q \) by \( \alpha h^m \), we obtain

\[ \frac{\partial h}{\partial t} + \frac{\partial (\alpha h^m)}{\partial x} \cos \theta + \frac{\partial (\alpha h^m)}{\partial y} \sin \theta = i - f \] (5)

which can be rearranged as

\[ \frac{\partial h}{\partial t} + m \alpha h^{m-1} \left( \cos \theta \frac{\partial h}{\partial x} + \sin \theta \frac{\partial h}{\partial y} \right) - i + f = 0 \] (6)

Providing value of rainfall intensity and the rate of water loss due to infiltration and evaporation, the finite element method can be used to solve for \( h \) at various time \( t \). Details are as follow:

Let the unknown variable \( h \) is approximated by \( \hat{h} \) which is a function of water depths at nodal points identified in the study domain as follows:

\[ \hat{h} = \sum_{i=1}^{n} N_i H_i = N^T H \] (7)

in which \( N_i \) is an interpolation function; \( H_i \) is water depth at node \( i \); \( N \) is matrix of \( N_i \); \( H \) is matrix of \( H_i \).

Replace \( h \) in Eq.(6) with \( \hat{h} \) will result in some error or residual \( R \), in which

\[ R = \frac{\partial \hat{h}}{\partial t} + m \alpha \hat{h}^{m-1} \left( \cos \theta \frac{\partial \hat{h}}{\partial x} + \sin \theta \frac{\partial \hat{h}}{\partial y} \right) - i + f \] (8)

In the weighted residual method, this residual is multiplied with a weighting function \( w \) and integral of their product over the whole study domain (\( \Omega \)) is set to zero. This results in the following weighted residual equation:

\[ \int_{\Omega} w \left( \frac{\partial \hat{h}}{\partial t} + m \alpha \cos \theta \frac{\partial \hat{h}}{\partial x} + m \alpha \sin \theta \frac{\partial \hat{h}}{\partial y} - i + f \right) \, dA = 0 \] (9)

The parameter \( \alpha \) and angle \( \theta \) usually vary depending on topography and land.
use of the area. In this study, the values $\alpha \cos \theta$ and $\alpha \sin \theta$ are expressed in terms of the values at nodal points using the same interpolation function. Let $\alpha_x = \alpha \cos \theta$ and $\alpha_y = \alpha \sin \theta$ be expressed in terms of their nodal values as follow:

$$\alpha_x = \alpha \cos \theta = \sum_{i=1}^{n} N_i \alpha_{xi} = N^T \alpha_x \text{ and } \alpha_y = \alpha \sin \theta = \sum_{i=1}^{n} N_i \alpha_{yi} = N^T \alpha_y \quad (10)$$

Substitute $\hat{h}, \alpha \cos \theta$ and $\alpha \sin \theta$ expressed in terms of their nodal values in Eq.(9), we obtain:

$$\int_{\Omega} \left[ \varepsilon \left( N^T H \right) + m N^T \alpha_x \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial x} \right] dA + \int_{\Omega} \left[ m N^T \alpha_y \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial y} - i + f \right] dA = 0 \quad (11)$$

In Galerkin’s method, the interpolation function $i (i = 1, 2, ..., n)$ used as the weighting function $w$. So, we obtain a set of $n$ weighted residual equations, which can be written in the matrix form as follows:

$$\int_{\Omega} N N^T dA \frac{\partial H}{\partial t} + \int_{\Omega} \left\{ m N N^T \alpha_x \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial x} \right\} dA + \int_{\Omega} \left\{ m N N^T \alpha_y \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial y} \right\} dA - \int_{\Omega} i N dA + \int_{\Omega} f N dA = 0 \quad (12)$$

which can be written in more compact form as:

$$M \frac{dH}{dt} + M_h - M_i + M_f = 0 \quad (13)$$

in which

$$M = \int_{\Omega} N N^T dA \quad (14)$$

$$M_h = \int_{\Omega} \left\{ m N N^T \alpha_x \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial x} \right\} dA + \int_{\Omega} \left\{ m N N^T \alpha_y \left( N^T H \right)^{m-1} \frac{\partial \left( N^T H \right)}{\partial y} \right\} dA \quad (15)$$

$$M_i = \int_{\Omega} i N dA \quad (16)$$

$$M_f = \int_{\Omega} f N dA \quad (17)$$

In finite element method, the study domain is divided into a number of elements and integral over the whole study domain is equal to the sum of integrals over these elements [10]. In this study, a linear triangular element is used for the kinematic overland flow model.

The Runge-Kutta method [12] is used to solve this set of first-order
differential equations. Once the water depth at each nodal point $H_i$ is known, the discharge per unit width at node $i$, $q_i$, can be computed from:

$$q_i = \alpha_i H_i^m = \frac{S_i}{n_{oi}} H_i^{\frac{m}{n}}$$  \hspace{1cm} (18)$$

When the overland flow rate per unit width $q_i$ at a nodal point located on the stream bank is obtained, the lateral inflow per unit length of stream $q_n$ can be determined from

$$q_n = q_i \left( \cos \theta \cos \beta + \sin \theta \sin \beta \right)$$  \hspace{1cm} (19)$$
in which $\theta$ is the angle between ground slope direction and the $x$ axis and $\beta$ is the angle between river flow direction and the $x$ axis as shown in Figure 1.

![Figure 1. Overland flow $q_i$ and lateral inflow $q_n$](image)

**Kinematic Stream Flow Model**

For stream channels, the basic equations for kinematic routing are [11]:

**Continuity equation:**

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_n - q_o$$  \hspace{1cm} (20)$$

**Discharge-area relationship:**

$$Q = \alpha_c A^{m_c}$$  \hspace{1cm} (21)$$
in which $Q$ is stream flow rate ($m^3/s$); $A$ is cross-sectional area ($m^2$); $q_n$ is lateral inflow ($m^3/s.m$); $q_o$ is rate of water loss due to evaporation and seepage ($m^3/s.m$); $\alpha_c, m_c$ are kinematic wave parameters of which the values depend on channel shape.

Replace $Q$ in Eq.(19) by $\alpha_c A^{m_c}$, we obtain:

$$\frac{\partial A}{\partial t} + \frac{\partial (\alpha_c A^{m_c})}{\partial x} = q_n - q_o$$  \hspace{1cm} (22)$$
or

$$\frac{\partial A}{\partial t} + m_c \alpha_c A^{m_c-1} \frac{\partial A}{\partial x} + A^{m_c} \frac{\partial \alpha_c}{\partial x} - q_n + q_o = 0$$  \hspace{1cm} (23)$$

Providing value of lateral inflow and seepage and other model parameters, the
finite element method can be used to solve for $A$ at various distance $x$ and time $t$. Details are as follow:

Let the unknown variable $A$ is approximated by a function of cross-sectional areas at nodal points identified in the study domain as follows:

$$\hat{A} = \sum_{i=1}^{n} N_i A_i = N^T A$$  \hspace{1cm} (24)

in which $N_i$ is an interpolation function; $A_i$ is cross-sectional area at node $i$; $N$ is matrix of $N_i$; $A$ is matrix of $A_i$.

Replace $A$ in Eq.(22) with $\hat{A}$ will result in some error or residual $R$ as follows:

$$R = \frac{\partial \hat{A}}{\partial t} + m_c \alpha_c \hat{A}^{m_c-1} \frac{\partial \hat{A}}{\partial x} + \hat{A}^{m_c} \frac{\partial \alpha_c}{\partial x} - q_n + q_o$$  \hspace{1cm} (25)

In weighted residual method, this residual is multiplied with a weighting function $w$ and integral of the product over the whole study domain ($\Omega$) is set to zero, which results in the following weighted residual equation:

$$\int_{\Omega} \left[ w \left( \frac{\partial \hat{A}}{\partial t} + m_c \alpha_c \hat{A}^{m_c-1} \frac{\partial \hat{A}}{\partial x} + \hat{A}^{m_c} \frac{\partial \alpha_c}{\partial x} - q_n + q_o \right) \right] dx = 0$$  \hspace{1cm} (26)

The parameter $\alpha_c$ usually varies with distance along the channel. In this study, it is expressed in terms of the nodal values using the same interpolation function as $\hat{A}$, that is

$$\alpha_c = \sum_{i=1}^{n} N_i \alpha_{ci} = N^\tau \alpha_c$$  \hspace{1cm} (27)

Substitute $\hat{A}$ and $\alpha_c$ in Eq.(25) by the expressions in Eqs.(23) and (26), we obtain:

$$\int_{\Omega} \left[ w \left( \frac{\partial (N^\tau A)}{\partial t} + m_c N^\tau \alpha_c (N^\tau A)^{m_c-1} \frac{\partial (N^\tau A)}{\partial x} 
+ (N^\tau A)^{m_c} \frac{\partial (N^\tau \alpha_c)}{\partial x} - q_n + q_o \right) \right] dx = 0$$  \hspace{1cm} (28)

In Galerkin's method, the interpolation function $\alpha_i$ $(i = 1,2,...,n)$ is used as the weighting function $w$. So, we obtain a set of $n$ weighted residual equations, which can be written in the matrix form as follows:

$$\int_{\Omega} NN^\tau dx \frac{\partial A}{\partial t} + \int_{\Omega} m_c NN^\tau \alpha_c (N^\tau A)^{m_c-1} \frac{\partial N^\tau A}{\partial x} A dx$$
$$+ \int_{\Omega} N (N^\tau A)^{m_c} \frac{\partial N^\tau A}{\partial x} \alpha_c dx - \int_{\Omega} q_n N dx + \int_{\Omega} q_o N dx = 0$$  \hspace{1cm} (29)

which can be written in more compact form as:
\[ M_I \frac{dA}{dt} + M_a + M_b - M_{qn} + M_{qo} = 0 \]  

(30)

in which

\[ M_I = \int_{\Omega} NN^T dx \]  

(31)

\[ M_a = \int_{\Omega} m_c NN^T \alpha_c (N^T A)^{m-1} \frac{\partial N^T}{\partial x} A dx \]  

(32)

\[ M_b = \int_{\Omega} N (N^T A)^m \frac{\partial N^T}{\partial x} \alpha_c dx \]  

(33)

\[ M_{qn} = \int_{\Omega} q_n N dx \]  

(34)

\[ M_{qo} = \int_{\Omega} q_o N dx \]  

(34)

In the finite element method, the study domain is divided into a number of elements and integral over the whole study domain is equal to the sum of integrals over these elements. In this study, a linear one-dimensional element is used for the kinematic stream flow model.

The Runge-Kutta method [12] is then used to solve this set of first-order differential equations. Once the cross-sectional area at each nodal point \( A_i \) is known, the discharge at node \( i \), \( Q_i \), can be computed from:

\[ Q_i = \alpha_{ci} A_i^m. \]  

(35)

**Model Application**

**Study Area**

The Wang river is one main tributary of the Chao Phraya river which is the most significant water resource of Thailand. The Wang river basin lies in the north-south direction, located between latitudes 16° 05' N. and 19° 30' N. and longitudes 98° 54' E. and 99°58' E. (Figure 2). The total catchment area covers about 10,791 km². The Wang river originates in Phi Pan Nam mountain range in Chiang Rai province and flows southwards passing Lampang and Tak provinces before joining the Ping river in Ban Tak district, Tak province. The total length of the Wang river is approximately 460 km. The Wang river basin is surrounded by mountain ranges along the eastern and western boundaries. Topography of the basin is characterized by narrow plains and valleys in the northern part and flood plains in the southern part [13].

Climate in the Wang river basin is influenced by Northeast and Southwest monsoons together with typhoons, monsoon troughs and depressions from South China Sea. The mean monthly temperature varies from 22.2 °C to 29.7 °C. The mean monthly rainfall varies from 4.9 mm to 243.3 mm with an annual average of 1,173.9 mm. The mean monthly evaporation varies from 90.3 mm to 186.3 mm with an annual average of 1,497.3 mm. The mean monthly relative humidity varies from 53.0% to 82.7% with an annual average of 72.7% [14].
Data Collection
Secondary data from various concerned agencies are used in model application and verification. These agencies include Royal Irrigation Department (RID) for stream flow data, Thai Meteorological Department (TMD) for climatic data, Land Development Department (LDD) for soil data, and Royal Thai Survey Department (RTSD) for topographic and land use data. A total of 13 rainfall stations and 5 runoff stations on the main river were used in this study. Locations of these rainfall and stream flow gauge stations are shown in Figure 3.

Figure 2. Wang river basin

Computation
At first, the kinematic overland flow model is applied to each sub-basin. The area is divided into a number of triangular elements. The overland flow rates at nodal points along the streams in each sub-basin at each time step are computed from the model and then the lateral inflows per unit length of the tributary streams in the sub-basin are determined. The initial overland water depth at each nodal point is set to zero. Also, water depths at all
nodal points located along the upper boundary of each sub-basin, which is mainly on the mountainous area, are set to zero at each time step. Size of time step is selected by trial and error so as to obtain stable results. The time step of 30 seconds is finally used in this study.

Next, the kinematic stream flow model is applied to all streams in each sub-basin. Each stream is divided into a number of one-dimensional elements. The results of lateral flow computation obtained from the overland flow model are used as input data to the kinematic stream flow model. Flow rates in the tributary streams at each time step are computed first.

Figure 3. Location of runoff stations and rainfall stations

Then, flow rates in the main streams are computed using the results of lateral discharges obtained from the overland flow model and the results of stream flows from the connected tributary streams as input data. For an tributary stream, the initial values of stream flow at all nodal points are set to zero, so as the value at the upstream end at each time step. For the main Wang river, data on river flow
recorded by RID on July 1, 2011 are used to interpolate initial flow at each node identified in the Wang river.

Model Calibration

Flow rates in the main streams are compared with the measured data obtained from the RID. Some parameters in the models, which include the rate of water loss due to infiltration and evaporation $f$, effective roughness parameter for overland flow $n_o$, and the rate of water loss due to evaporation and seepage $q_o$, are adjusted so that the results from the models agree with the measured data.

Results and Discussions

The model parameters at 5 runoff stations are used for the overland flow and channel flow computation. After applying these parameters together with the rainfall data in the model, the simulation results are compared with the observed data at 5 runoff stations as shown in Figures 4-8. For correlation analysis, three statistical indices are determined, these include coefficient of determination, $R^2$, Nash–Sutcliffe model efficiency coefficient, NSE, and coefficient of variation of the root mean square error, CV(RMSE). The obtained results are summarized in Table 1.

It is found that the values of $R^2$ and NSE at stations W.10A and W.5A are higher than 0.70 and the values of CV(RMSE) are less than 0.50, indicating that the simulation results are good fitted with the observed data. So, the simulation results at these two stations are acceptable. For stations W.1C, the values of $R^2$ and CV(RMSE) are moderate, but the value of NSE is rather low, indicating that there are remarkable differences between the simulation results and the observed data. For stations W.6A and W.4A, the values of $R^2$ and NSE are rather low, while the values of CV(RMSE) are rather high, indicating that there are significant differences between the simulation results and the observed data.

The results obtained from this study show that at some stations the difference between the model results and the measured data can be noticed. These are due to some factors, including: 1) there are only few rain gauge stations in the study area, so the rainfall data applied to each element in the watershed might be different from the real rainfall patterns; 2) topography of the upstream watershed is mountainous with steep slope, so, it is rather difficult to determine the slope of the area in each element from the available topographic maps; 3) land use maps of the study area are not up-to-date; and 4) data on cross-sectional areas of various streams are available at some sections only, so interpolation has been used.
Figure 4. Simulated and observed flood hydrograph at runoff station W.10A

Figure 5. Simulated and observed flood hydrograph at runoff station W.1C
Figure 6. Simulated and observed flood hydrograph at runoff station W.5A

Figure 7. Simulated and observed flood hydrograph at runoff station W.6A
for estimating the value of cross-sectional area at each node identified in the model; 5) it is rather difficult to estimate the rate of water loss due to evaporation and seepage ($q_o$) in the stream flow model. Error in assuming the values of $q_o$ in various elements in the simulated model will lead to error in stream flow rates computed from the model.

**Conclusions**

In this study, mathematical equations expressing overland flow and stream flow are used as basis equations in developing two-dimensional overland flow model and one-dimensional stream flow model. The finite element method with Galerkin’s weighted residual technique is used in model formulation. The second-order Runge-Kutta method is applied to solve the set of differential equations obtained from finite element formulation. The developed models are applied to the Wang river basin in the northern region of Thailand during July 1 - October 31, 2011 when severe flooding occurred in this river basin.

In comparing the model results with the observed river flow data of RID at 5 gauging
stations in the Wang river, it is found that the model results at the upstream portion of the river (the upper 3 stations) satisfactorily agree with the observed data, with the values of $R^2$ greater than 0.55 and CV(RMSE) less than 0.57, though the NSE value at station W.1C is rather low. For the downstream portion of the river, there are remarkable differences between the model results and the observed data. The values of $R^2$ are less than 0.35, CV(RMSE) greater than 0.76, and the NSE values are less than 0.16. This might be due to some errors in the input data, including rainfall pattern, topography, land use, river cross-sectional area, and water seepage along the river. More detailed field investigation and model calibration are still needed.

Acknowledgments

The authors gratefully appreciate financial support from Kasetsart University and the National Research Council of Thailand (NRCT) and the Japan Science Promotion Society (JSPS). They are grateful to the Royal Irrigation Department, the Thai Meteorological Department, the Land Development Department, and the Royal Thai Survey Department for providing useful data for this study.

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