$$A_{22} = \frac{1}{3} m_2 {l_2}^2 \tag{6}$$

B is a $n \times n$ matrix that represents the Coriolis force at the first link due to the velocity at the second link. It is happened due to the first link act as the rotating frame for the second link (Niku, 2011).

$$[B] = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{7}$$

$$B_{11} = -m_2 l_1 l_2 sin\theta_2 \tag{8}$$

$$B_{12} = B_{21} = B_{22} = 0 \tag{9}$$

C is described below as a $n \times n$ matrix is a centripetal term caused by the centrifugal effect.

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{10}$$

$$C_{11} = C_{21} = 0 \tag{11}$$

$$C_{12} = -\frac{1}{2} m_2 l_1 l_2 sin\theta_2 \tag{12}$$

$$C_{22} = -C_{12} \tag{13}$$

D is a $n \times m$ matrix related to the gravitational acceleration,

$$[D] = \begin{bmatrix} D_{11} \\ D_{12} \end{bmatrix} \tag{14}$$

$$D_{11} = \left(\frac{1}{2} m_1 + m_2\right) g l_1 cos\theta_1 + \frac{1}{2} m_2 g l_2 cos\theta_{12} \tag{15}$$

$$D_{12} = \frac{1}{2} m_2 g l_2 cos\theta_{12} \tag{16}$$

Linear PID Control System

The PID control law in its standard form is in Equation 17 [5].

$$\tau(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int_0^t e(t) dt \tag{17}$$

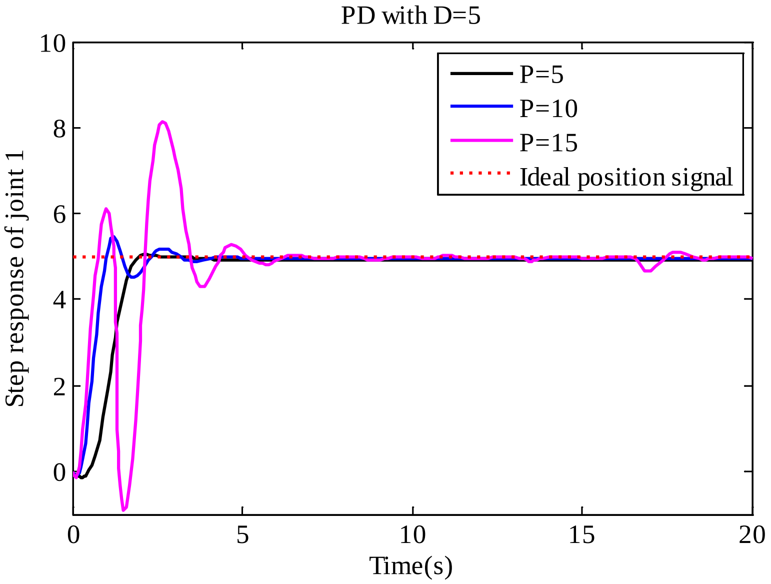
K_P,$K_I$and $K_D$is the proportional, integrator and derivative gain respectively, $e(t) = \theta_d - \theta$ is the tracking error of the system.

The performance of PID control system strictly relied on the value of gain K_P, $K_I$and K_D. The gain of PID controller is tuned based on the individual effect of the three terms in closed loop performance and can be referred to[6]. However, for this case a combination of PID gain modification will be considerate. The K_P gain is vary from 5, 10 and 15,$K_I$gain vary from 0 to 5 and K_D gain is vary from 5, 10 and 15.

Computed Torque Control (CTC) System

Computed torque control (CTC) which consist of two main parts the feed-forward and the feedback component. The feed-forward component is a nonlinear compensation provides the amount of torque required to drive the system along its nominal path. On the other hand, the feedback component provided a corrective torque to reduce any error along the trajectory of manipulator. PID computed torque control (PIDCTC) is one of the controllers which implemented the PID as the feedback component.

Based on Figure 2, increasing K_P, improved the steady state error. However, it caused overshoot to increase. Since the overshoot can be compensated by derivative term, as will be discussed in the next paragraph, we will choose higher gain of K_P as 15.



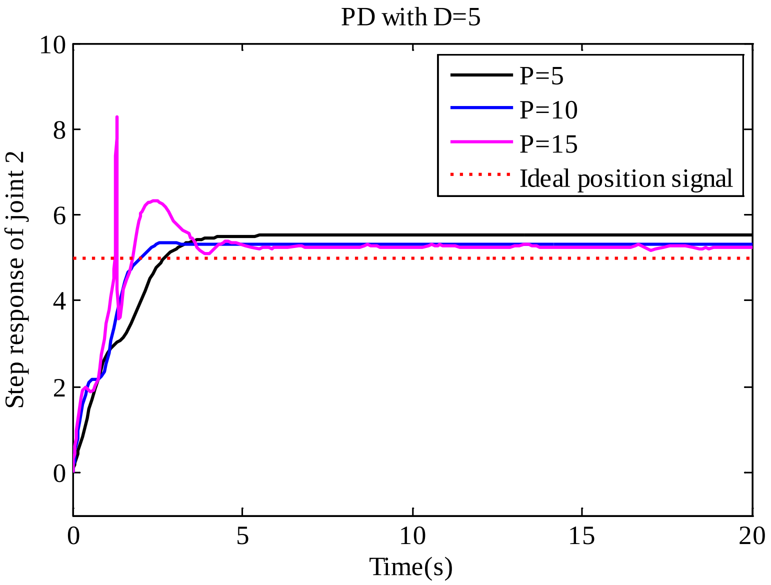


Figure 2. Step response PD for varyingK_P

Referring to Figure 3, the effect of increasing $K_D$when K_P is 15, caused reductions in overshoot and steady state error compare to K_D = 5 and 10. Thus, K_D is chosen as 15.

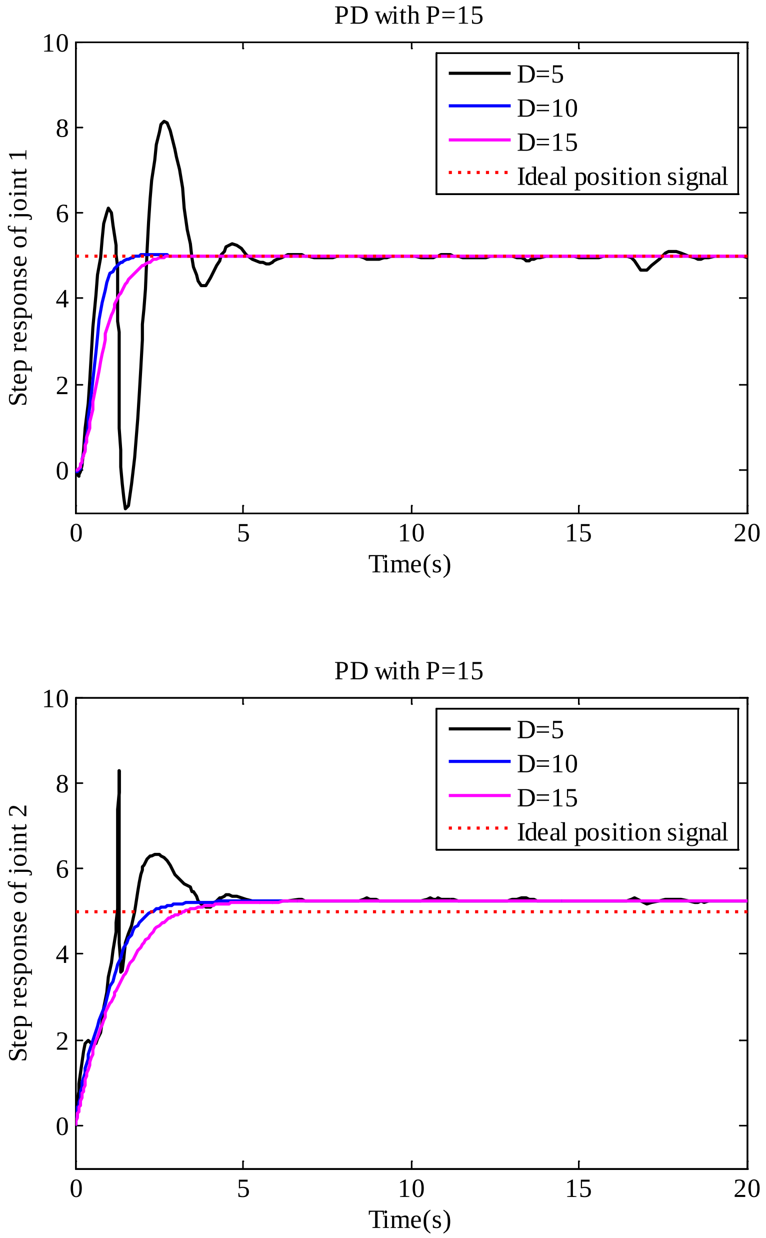
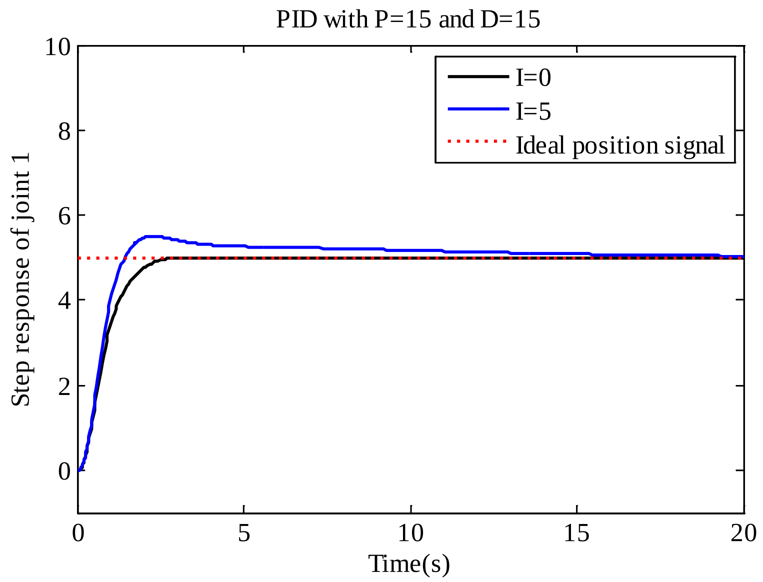


Figure 3. Step response PD for varying K_D

Based on the above discussion, setting K_P and K_D as 15, the effect of installing integral term, $K_I = 5$, improved the steady state error especially for joint 1. However it negatively affects the overshoot of the system as illustrated in Figure 4.



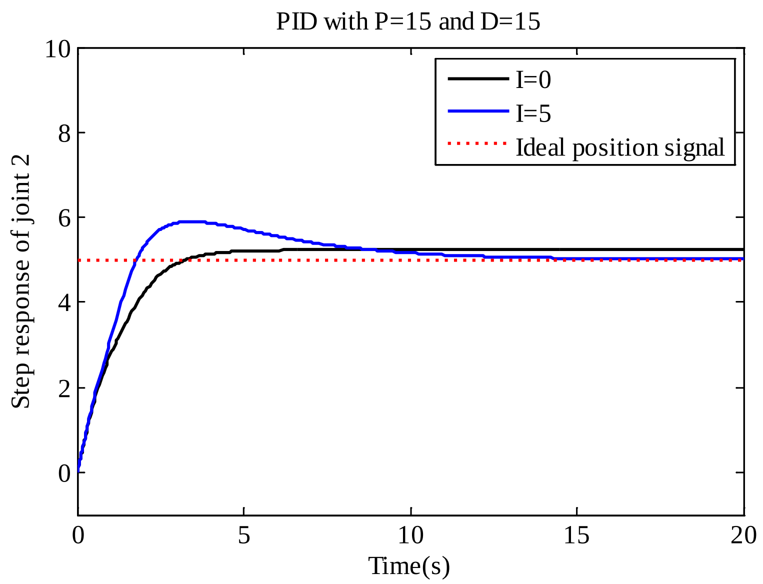


Figure 4. Step response PID for varyingK_I

Thus the combination of PID gain is chosen as, $K_P = 15$, $K_I = 5$ and $K_D = 15$ or $K_P = 15$, $K_I = 0$ and $K_D = 15$. The selection between such gains depends on the requirement of the system. For the system which can tolerate the overshoot but require high accuracy, the PID is a better choice. However, if the accuracy is not the matter to be concern, PD is better choice. The result obtained from PID controller is a trade-off between steady state error and overshoot.

The PID gain in PIDCTC does not follow completely the condition in [6]. Table 3 shows the performance of varying the PIDCTC gains. Varying K_P gain is shown in row 1, 2 and 3. Using the lowest K_P = 20, yield the highest PO for joint 1. Increasing K_P to 60 reduced the PO to 0.15. Further increase to 100, slightly increase the PO to 0.16. However, using the highest K_P= 100, produced the lowest SSE. Thus higher proportional gain is better. $K_P$is chosen as 100.

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Row	Variable			Percent Overshoot, PO (%)		Steady State Error, SSE (%)	
	K_P	K_I	K_D				
1	20	0	20	1.33	0.00	1.58	14.06
2	60	0	20	0.15	0.01	1.06	5.00
3	100	0	20	0.16	0.03	0.81	2.82
4	100	0	60	0.01	0.00	0.78	2.77
5	100	0	100	0.00	0.00	0.76	2.71
6	100	0	20	0.16	0.03	0.81	2.82
7	100	20	20	3.80	6.95	0.78	0.53
8	100	60	20	9.06	13.37	0.21	0.12
9	100	100	20	13.73	18.07	0.11	0.07

The effect of varying K_D gain is shown in row 3, 4 and 5. Increasing K_D give a similar pattern as in [6]. PO reduced as increased the K_D gain. There is slightly no change in steady state error for both joints. However, considering the settling time of the system, as increasing the K_D gain, increased the time taken for the system to reach the steady state from 0.53s to 3.42s and 0.72s to 4.50s for joint 1 and joint 2 respectively. Thus, K_D gain is chosen as 20.

Row 6, 7, 8 and 9 shows the effect of varying K_I from 20, 60 to 100. The obvious effect is increasing the percent overshoot as increasing the K_I gain. However, it improved the SSE for joint 1.

Thus the combination of PIDCTC gain can be chosen as $K_P = 100$,$K_I = 100$and $K_D = 20$ or $K_P = 100$, $K_I = 0$ and $K_D = 20$. The selection between such gains shared the similar concept as PID controller.

The result SMCTC for varying c and k shows the SMC exhibit virtually no steady state error and overshoot. Thus for choosing the best gain for the system, the fast reaching mode is considered. Lower$c$ value and higher k provided the fastest reaching mode. Thus, c and k is chosen as 20 and 100 respectively.

Tracking Performance of Controllers

Table 4 shows the comparison of tracking performance between the three controllers without considering the presence of disturbance and in the presence of disturbance. PIDCTC is better tracker than PID controller. However, when the system imposed to an external disturbance, the performance of PIDCTC is degraded as the tracking error increase drastically. While, SMCTC performance is not affected by disturbance.

Table 4. Comparison of Tracking Error

Control	Joint	Average Tracking Error	
		No Disturbance	With Disturbance
PID	1	0.29	1.05
	2	0.14	0.91
CTC	1	0.05	0.13
	2	0.04	0.53
SMC	1	0.00	0.00
	2	0.00	0.00

As shown in Figure 5, the result for SMCTC is overlapped with the ideal position track. That means the system is able to track the ideal position perfectly compared to PIDCTC and PID controller with tracking error explicitly lowest among all controllers.

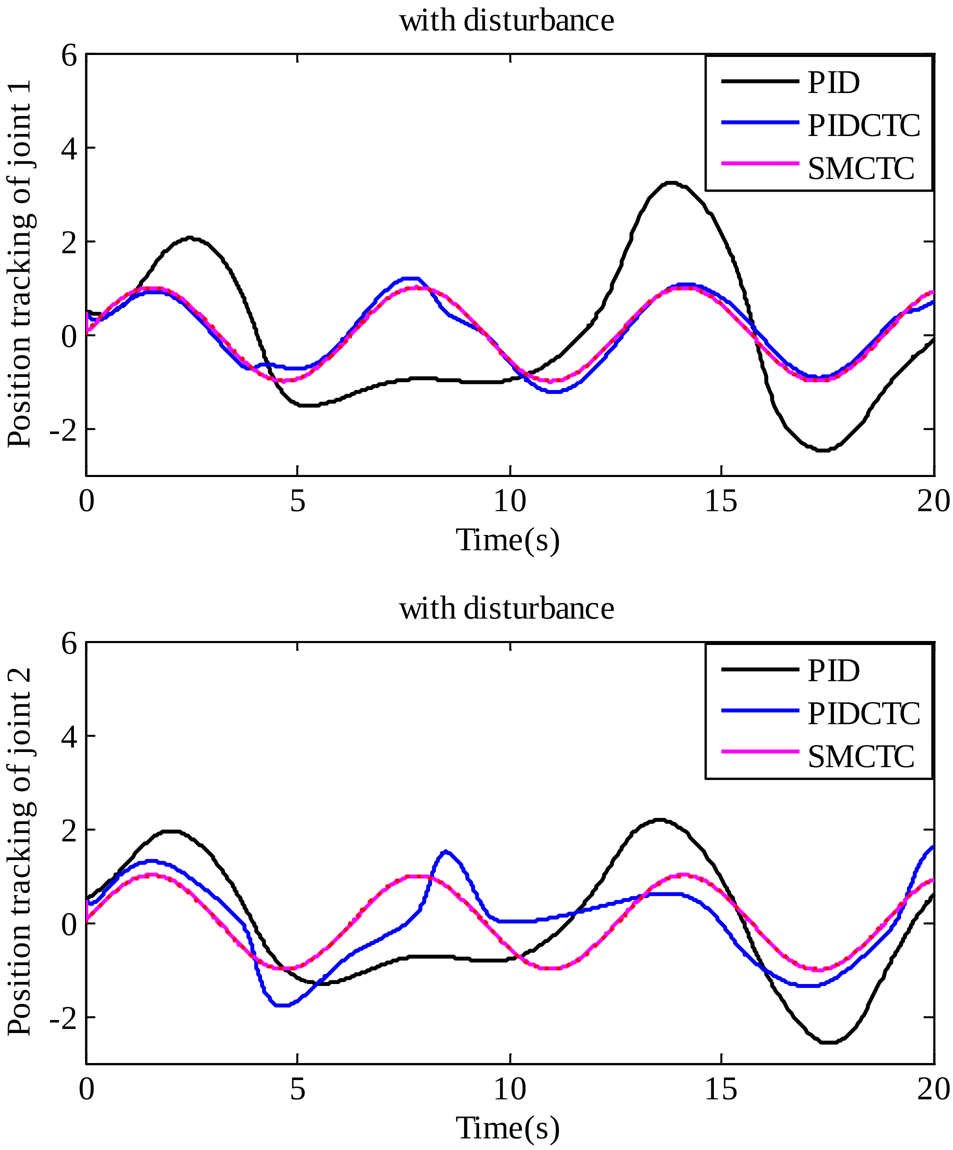

Figure 5. Comparison position tracking for PID, CTC and SMC with disturbance

Conclusions

For the proposed robot manipulator system, the PID gain can be chosen as $K_P = 15, K_I = 5$ and $K_D = 15$ or $K_P = 15, K_I = 0$ and $K_D = 15$ and the PIDCTC gain are $K_P = 100, K_I = 100$ and $K_D = 20$ or $K_P = 100, K_I = 0$ and $K_D = 20$. The result shows that both controllers unable to performed the lowest overshoot and steady state error for a single gain configuration. However, PIDCTC shows a better tracking performance than PID. But in real application, the robot experienced unknown external disturbance. When installing disturbance in the system, SMCTC is superior in tracking control than PIDCTC and PID. It is proven that the SMC is a robust control system.

For further improvement, model estimation should be considered especially for the controller system required precise knowledge of system modeling. This is essential for controller verification that will be tested on servo motor robot manipulator.

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